Received: *November 07, 2000*, Accepted: *January 12, 2001*

The action of $N = 4$ Super Yang-Mills from a chiral primary operator

Vitor E.R. Lemes, Silvio P. Sorella and Ozemar S. Ventura*

UERJ, Universidade do Estado do Rio de Janeiro Rua São Francisco Xavier 524, 20550-013 Maracanã, Rio de Janeiro, Brazil
E-mail: vitor@dft.if.uerj.br, sorella@dft.if.uerj.br, ozemar@dft.if.uerj.br

Marcelo S. Sarandy

CBPF, Centro Brasileiro de Pesquisas Fśicas Rua Xavier Sigaud 150, 22290-180 Urca, Rio de Janeiro, Brazil
E-mail: sarandy@cbpf.br

Alessandro Tanzini

Dipartimento di Fisica, Universitá di Roma "Tor Vergata" Via della Ricerca Scientifica 1, 00173 Roma, Italy
E-mail: tanzini@roma2.infn.it

Abstract: Using the Vafa-Witten twisted version of $N = 4$ super Yang-Mills a subset of the supercharges actually relevant for the non-renormalization properties of the theory is identified. In particular, a relationship between the gauge-fixed action and the chiral primary operator $\mathrm{tr}\,\phi^2$ is worked out. This result can be understood as an off-shell extension of the reduction formula introduced by Intriligator in [1].

Keywords: BRST Quantization, AdS/CFT Correspondance, Extended Supersymmetry.

*Fundação Osório, Rua Paula Ramos 52,20264-210 Rio Comprido, Rio de Janeiro, Brazil

JHEP01(2001)016

Contents

1. Introduction

It is a well established fact that the $N = 4$ super Yang-Mills (SYM) theory in four dimensions displays a set of remarkable properties, both at perturbative and non-perturbative level. Its β-function is found to vanish to all orders of perturbation theory [2, 3]: a feature understood as a consequence of the absence of anomalies for the super-conformal invariance [3]. Also, the theory is believed to exhibit an exact electromagnetic duality [4], not destroyed by the quantum corrections.

More recently, the conjectured AdS/CFT correspondence [5] between type-IIB superstring on $AdS_5 \times S^5$ and $N = 4$ SYM theory has renewed the interest on the finiteness properties of this theory in its superconformal phase. In fact, many of the tests of the AdS/CFT conjecture have relied on non-renormalization properties, which are crucial in order to ensure a meaningful comparison between the strong coupling regime, accessible by type-IIB supergravity computations, and the weak coupling one, where the field theory techniques are reliable. In particular, it has been pointed out [6] that a whole class of certain n-point correlation functions of chiral primary operators should obey non-renormalization theorems. The analysis of these correlators provides thus a highly non-trivial check for the AdS/CFT correspondence, being, at present, object of intensive research [7, 8, 9].

In this work we study the $N = 4$ SYM by using the Vafa-Witten twisted version [10]. As it is well known, when the model is formulated on $\mathbb{R}^4$, the twist simply amounts to a linear change of variables, so that the twisted theory is completely equivalent to the conventional one. Nonetheless, the use of the twisted variables considerably simplifies the analysis of the finiteness properties, allowing to identify a subset of supercharges which is actually relevant to control the ultraviolet behavior.

Moreover, the combined use of the Wess-Zumino gauge and of the Batalin-Vilkovisky procedure will allow us to obtain a generalized BRST operator which encodes all relevant generators of $N = 4$, yielding an algebraic off-shell characterization of the fully quantized action.

In particular, we shall be able to establish an interesting relationship between the gauge-fixed $N = 4$ SYM action and the chiral primary operator tr ϕ^2, where in the $N = 2$ formalism ϕ corresponds to the scalar field of the vector multiplet. This gauge invariant polynomial can be regarded as a kind of perturbative prepotential for the $N = 4$ SYM action, which is in fact obtained from tr ϕ^2 by repeated application of the twisted generators of the $N = 4$ superalgebra.

This result can be understood as an off-shell extension of the reduction formula discussed for the first time in [1][1] and exploited in [8, 9] in order to analyse the non-renormalization properties of the above mentioned correlators.[2]

We remark that an analogous relationship has been also proven to hold in the case of $N = 2$ SYM theory [11], where it has been possible to give a complete algebraic proof of the celebrated non-renormalization theorem of the $N = 2$ beta function by making use of the vanishing of the anomalous dimension of tr ϕ^2. This property is maintained in the case of $N = 4$, providing an elementary proof of the ultraviolet finiteness of the theory.

The organization of the paper is as follows. In section 2 we briefly review the twisting procedure of Vafa-Witten. In section 3 the quantum extension of the theory is provided within the antifield formalism. The section 4 is devoted to the relationship between tr ϕ^2 and the action of $N = 4$. In section 5 we summarize our main results, presenting the conclusions.

2. The twisted $N = 4$ super Yang-Mills theory

The global symmetry group of $N = 4$ SYM theory in euclidean space-time is $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(4)$, where $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ is the rotation group and SU(4) the internal symmetry group of $N = 4$. Hence the twist operation can be performed in more than one way, depending on how the internal symmetry group is combined with the rotation group [12]. In the case of the twist proposed by Vafa and Witten [10] the SU(4) is splitted as $\mathrm{SU}(2)_F \times \mathrm{SU}(2)_I$, so that the twisted global symmetry group turns out to be $\mathrm{SU}(2)'_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(2)_F$, where $\mathrm{SU}(2)'_L = \mathrm{diag}(\mathrm{SU}(2)_L \oplus \mathrm{SU}(2)_I)$ and $\mathrm{SU}(2)_F$ is a residual internal symmetry group. The fields of the $N = 4$ multiplet are given by $(A_\mu, \lambda^\alpha_u, \overline{\lambda}^u_{\dot\alpha}, \Phi_{uv})$, where $(u, v = 1, \ldots, 4)$ are indices of the fundamental representation of SU(4), and the six real scalar fields of the model are collected into the antisymmetric and self-conjugate tensor Φ_{uv}. Under the twisted group, these

[1] See eq. (5.5) of hep-th/9811047.

[2] Observe that an off-shell version of the reduction formula has been obtained by [9] in the framework of the $N = 2$ harmonic superspace formalism for $N = 4$ SYM.

fields decompose as

$$
\begin{aligned}
A_\mu &\longrightarrow A_\mu\,, \\
\overline{\lambda}^u_{\dot\alpha} &\longrightarrow \psi^i_\mu\,, \\
\lambda^\alpha_u &\longrightarrow \eta^i,\ \chi^i_{\mu\nu}\,, \\
\Phi_{uv} &\longrightarrow B_{\mu\nu},\ \phi^{ij}\,,
\end{aligned}
\tag{2.1}
$$

where $(i,j=1,2)$ are indices of the residual isospin group $\mathrm{SU}(2)_F$, ϕ^{ij} is a symmetric tensor, and $\chi^i_{\mu\nu}$, $B_{\mu\nu}$ are self-dual with respect to the Lorentz indices. Since in our analysis manifest isospin invariance is not needed, we further explicit the $\mathrm{SU}(2)_F$ doublets as $\psi^i_\mu = (\psi_\mu, \chi_\mu)$, $\eta^i = (\eta, \xi)$, $\chi^i_{\mu\nu} = (\chi_{\mu\nu}, \psi_{\mu\nu})$ and the triplet as $\phi^{ij} = (\phi, \overline{\phi}, \tau)$. As we shall see later on, the subset $(A_\mu, \psi_\mu, \chi_{\mu\nu}, \eta, \phi, \overline{\phi})$ can be readily recognized as the twisted vector multiplet of $N=2$ [13].

Following [14], the action of $N=4$ SYM in terms of the twisted variables is found to be

$$
\begin{aligned}
S_{N=4} = \frac{1}{g^2}\,\mathrm{tr}\int d^4x\,\Big(&D_\mu\phi D^\mu\overline{\phi} + i\psi_\mu D_\nu\chi^{\mu\nu} + i\chi_\mu D_\nu\psi^{\mu\nu} - \chi_\mu D^\mu\xi + \\
&+\ \psi_\mu D^\mu\eta - i\overline{\phi}\left\{\psi_{\mu\nu},\psi^{\mu\nu}\right\} + i\phi\left\{\chi_{\mu\nu},\chi^{\mu\nu}\right\} + i\tau\left\{\psi_{\mu\nu},\chi^{\mu\nu}\right\} - \\
&-\ \left\{\psi_{\mu\nu},\chi^{\mu\rho}\right\}B_\rho{}^{\nu} - i\chi_{\mu\nu}\left[\xi,B^{\mu\nu}\right] - i\psi_{\mu\nu}\left[\eta,B^{\mu\nu}\right] + 4i\overline{\phi}\left\{\xi,\xi\right\} - \\
&-\ 4i\,\phi\,\left\{\eta,\eta\right\} + 4i\,\tau\,\left\{\xi,\eta\right\} + \psi_\mu\left[\chi_\nu,B^{\mu\nu}\right] + i\,\phi\,\left\{\chi_\mu\,,\chi^\mu\right\} - \\
&-\ i\overline{\phi}\left\{\psi_\mu,\psi^\mu\right\} - i\psi_\mu\left[\chi^\mu,\tau\right] - 4\left[\phi,\overline{\phi}\right]\left[\phi,\overline{\phi}\right] + 4\left[\phi,\tau\right]\left[\overline{\phi},\tau\right] + \\
&+\ \left[\phi\,,B_{\mu\nu}\right]\left[\overline{\phi}\,,B^{\mu\nu}\right] - H^\mu\left(H_\mu - D_\mu\tau + i\,D^\nu B_{\mu\nu}\right) + \\
&+\ H^{\mu\nu}\left(-H^+_{\mu\nu} + \frac{i}{4}F^+_{\mu\nu} - \frac{1}{2}\left[B_{\mu\rho},B^\rho{}_\nu\right] - i\left[B_{\mu\nu},\tau\right]\right)\Big)\,,
\end{aligned}
\tag{2.2}
$$

where g is the unique coupling constant and $H_{\mu\nu}$, H_μ are auxiliary fields, with $H_{\mu\nu}$ self-dual and $F^+_{\mu\nu} = F_{\mu\nu} + \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$.

Notice that in this formulation the invariance under the Cartan subgroup of $\mathrm{SU}(2)_F$ is still preserved, so that we can define a conserved charge, called here G-charge. The G-charges of all fields as well as their canonical dimensions and statistics[3] are displayed in the following table.

field	A_μ	ξ	η	$\psi_{\mu\nu}$	$\chi_{\mu\nu}$	ψ_μ	χ_μ	ϕ	τ	$\overline{\phi}$	$B_{\mu\nu}$	H_μ	$H_{\mu\nu}$
G-ch.	0	1	-1	1	-1	1	-1	2	0	-2	0	0	0
dim.	1	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	1	1	1	1	2	2
nature	c	a	a	a	a	a	a	c	c	c	c	c	c

Table 1: G-charge, canonical dimension and nature of the fields.

[3] The statistics (nature) of the fields is determined by their G-charges. Even values of the G-charge correspond to commuting (c) fields and odd values to anticommuting (a) fields.

We observe that, for the fields $(A_\mu, \psi_\mu, \chi_{\mu\nu}, \eta, \phi, \overline{\phi})$, the G-charge coincides with the R-charge of $N = 2$ [13].

The action (2.2) is invariant under gauge transformations with infinitesimal parameter ζ:

$$
\begin{aligned}
\delta^g_\zeta A_\mu &= -D_\mu \zeta = -\left(\partial_\mu \zeta + i\,[A_\mu, \zeta]\right), \\
\delta^g_\zeta \gamma &= i\,[\zeta, \gamma] \qquad \text{with} \qquad \gamma = \Big(\phi, \overline{\phi}, \psi_\mu, \psi_{\mu\nu}, \chi_\mu, \chi_{\mu\nu}, \xi, \eta, B_{\mu\nu}, \tau, H_\mu, H_{\mu\nu}\Big).
\end{aligned}
\tag{2.3}
$$

Concerning the generators $(\delta^\alpha_u, \overline{\delta}^u_{\dot\alpha})$ of the $N = 4$ superalgebra, it turns out that the twisting procedure gives rise to the following twisted charges [14]: two scalars, δ^+ and δ^-, two vectors, δ^+_μ and δ^-_μ, and two self-dual tensors $\delta^+_{\mu\nu}$ and $\delta^-_{\mu\nu}$. Of course, all twisted generators leave the action (2.2) invariant. In particular, the action of the scalar generator δ^+ is

$$
\begin{aligned}
\delta^+ A_\mu &= \psi_\mu\,, \qquad \delta^+ \tau = \xi\,, \\
\delta^+ \psi_\mu &= D_\mu \phi\,, \qquad \delta^+ \chi_\mu = H_\mu\,, \\
\delta^+ \phi &= 0\,, \qquad \delta^+ \xi = i\,[\tau, \phi]\,, \\
\delta^+ \overline{\phi} &= -\eta\,, \qquad \delta^+ B_{\mu\nu} = \psi_{\mu\nu}\,, \\
\delta^+ \eta &= i[\phi, \bar{\phi}], \qquad \delta^+ \psi_{\mu\nu} = i[B_{\mu\nu}, \phi]\,, \\
\delta^+ \chi_{\mu\nu} &= H_{\mu\nu}\,, \qquad \delta^+ H_\mu = i\,[\chi_\mu, \phi]\,, \\
\delta^+ H_{\mu\nu} &= i\,[\chi_{\mu\nu}, \phi]\,.
\end{aligned}
\tag{2.4}
$$

As anticipated, in the first column of eq. (2.4) we recognize the scalar transformations of the twisted $N = 2$ subalgebra [13], in presence of the auxiliary field $H_{\mu\nu}$. For δ^- one gets

$$
\begin{aligned}
\delta^- A_\mu &= \chi_\mu\,, \qquad \delta^- \tau = -\eta\,, \\
\delta^- \chi_\mu &= -D_\mu \overline{\phi}\,, \qquad \delta^- \psi_\mu = -H_\mu + D_\mu \tau\,, \\
\delta^- \bar{\phi} &= 0\,, \qquad \delta^- \eta = i[\tau, \bar{\phi}], \\
\delta^- \phi &= -\xi\,, \qquad \delta^- \chi_{\mu\nu} = i[B_{\mu\nu}, \bar{\phi}], \\
\delta^- \xi &= i[\phi, \overline{\phi}], \qquad \delta^- B_{\mu\nu} = -\chi_{\mu\nu}\,, \\
\delta^- \psi_{\mu\nu} &= H_{\mu\nu} + i\,[B_{\mu\nu}, \tau]\,, \\
\delta^- H_{\mu\nu} &= -i[\psi_{\mu\nu}, \bar{\phi}] + i\,[\chi_{\mu\nu}, \tau] + i\,[B_{\mu\nu}, \eta]\,, \\
\delta^- H_\mu &= -D_\mu \eta + i[\psi_\mu, \bar{\phi}] + i\,[\chi_\mu, \tau]\,.
\end{aligned}
\tag{2.5}
$$

Analogously, for the vector transformations δ^+_μ and δ^-_μ one obtains

$$
\begin{aligned}
\delta^+_\mu A_\nu &= -4i\chi_{\mu\nu} - 4g_{\mu\nu}\eta\,, \qquad \delta^+_\mu \tau = \chi_\mu\,, \\
\delta^+_\mu \phi &= \psi_\mu\,, \qquad \delta^+_\mu \bar{\phi} = 0\,,
\end{aligned}
$$

$$
\begin{aligned}
\delta^+_\mu \xi &= D_\mu \tau - H_\mu\,, \qquad \delta^+_\mu \eta = -D_\mu \overline{\phi}\,, \\
\delta^+_\mu B_{\nu\rho} &= -i\theta_{\mu\nu\rho\lambda}\chi^\lambda\,, \qquad \delta^+_\mu \psi_{\nu\rho} = D_\mu B_{\nu\rho} + i\theta_{\mu\nu\rho\lambda}H^\lambda\,, \\
\delta^+_\mu \chi_{\nu\rho} &= i\theta_{\mu\nu\rho\lambda}D^\lambda\overline{\phi}\,, \qquad \delta^+_\mu \chi_\nu = -4\left[B_{\mu\nu},\overline{\phi}\right] + 4ig_{\mu\nu}\left[\tau,\overline{\phi}\right]\,, \\
\delta^+_\mu \psi_\nu &= 4iH_{\mu\nu} + F_{\mu\nu} - 4ig_{\mu\nu}[\,\overline{\phi},\phi]\,, \\
\delta^+_\mu H_{\nu\rho} &= D_\mu\chi_{\nu\rho} + \theta_{\mu\nu\rho\lambda}\left[\psi^\lambda,\overline{\phi}\right] + i\theta_{\mu\nu\rho\lambda}D^\lambda\eta\,, \\
\delta^+_\mu H_\nu &= D_\mu\chi_\nu + 4\left[\eta,B_{\mu\nu}\right] + 4\left[\psi_{\mu\nu},\overline{\phi}\right] - 4ig_{\mu\nu}\left[\eta,\tau\right] - 4ig_{\mu\nu}\left[\xi,\overline{\phi}\right]\,, \qquad (2.6)
\end{aligned}
$$

and

$$
\begin{aligned}
\delta^-_\mu A_\nu &= -4i\psi_{\mu\nu} + 4g_{\mu\nu}\xi\,, \qquad \delta^-_\mu \tau = \psi_\mu\,, \\
\delta^-_\mu \phi &= 0\,, \qquad \delta^-_\mu\,\overline{\phi} = -\chi_\mu\,, \\
\delta^-_\mu \xi &= -D_\mu\phi\,, \qquad \delta^-_\mu \eta = -H_\mu\,, \\
\delta^-_\mu B_{\nu\rho} &= +i\theta_{\mu\nu\rho\lambda}\psi^\lambda\,, \qquad \delta^-_\mu \psi_\nu = -4\left[B_{\mu\nu},\phi\right] - 4ig_{\mu\nu}\left[\tau,\phi\right]\,, \\
\delta^-_\mu \psi_{\nu\rho} &= -i\theta_{\mu\nu\rho\lambda}D^\lambda\phi\,, \\
\delta^-_\mu \chi_{\nu\rho} &= -D_\mu B_{\nu\rho} - i\theta_{\mu\nu\rho\lambda}H^\lambda + i\theta_{\mu\nu\rho\lambda}D^\lambda\tau\,, \\
\delta^-_\mu \chi_\nu &= 4iH_{\mu\nu} + F_{\mu\nu} + 4ig_{\mu\nu}\left[\overline{\phi},\phi\right] - 4\left[B_{\mu\nu},\tau\right]\,, \\
\delta^-_\mu H_{\nu\rho} &= D_\mu\psi_{\nu\rho} + \theta_{\mu\nu\rho\lambda}\left(\left[\psi^\lambda,\tau\right] - \left[\chi^\lambda,\phi\right] - iD^\lambda\xi\right) + i\left[\psi_\mu,B_{\nu\rho}\right]\,, \\
\delta^-_\mu H_\nu &= -D_\mu\psi_\nu + D_\nu\psi_\mu + 4\left[\psi_{\mu\nu},\tau\right] - 4\left[\xi,B_{\mu\nu}\right] + 4\left[\chi_{\mu\nu},\phi\right] + 4ig_{\mu\nu}\left[\eta,\phi\right]\,. \qquad (2.7)
\end{aligned}
$$

where $\theta_{\mu\nu\rho\sigma}$ denotes the combination

$$
\theta_{\mu\nu\rho\sigma} = \varepsilon_{\mu\nu\rho\sigma} + g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\nu\sigma} \; = 4\Pi^+_{\mu\sigma\nu\rho}\,, \qquad (2.8)
$$

where $\Pi^+_{\mu\sigma\nu\rho}$ is the projector on self-dual two-forms.

It is worth emphasizing that the invariant action $S_{N=4}$ is uniquely fixed by the two vector generators δ^+_μ, δ^-_μ and by the scalar charge δ^+. In other words, the requirement of invariance under δ^+_μ, δ^-_μ and δ^+ fixes all the relative numerical coefficients of the various terms of the action (2.2). Due to this property, the tensorial transformations $\delta^+_{\mu\nu}$, $\delta^-_{\mu\nu}$ will not be taken into further account, although their inclusion can be done straightforwardly. Notice that a similar situation has already been met in the case of $N=2$ [15].

Let us give here the algebraic relations among the twisted generators, i.e.

$$
\begin{aligned}
\left\{\delta^+,\delta^+\right\} &= \delta^g_{-2\phi}\,, \qquad \left\{\delta^+_\mu,\delta^+\right\} = \partial_\mu + \delta^g_{A_\mu}\,, \\
\left\{\delta^-,\delta^-\right\} &= \delta^g_{2\overline{\phi}}\,, \qquad \left\{\delta^-_\mu,\delta^-\right\} = \partial_\mu + \delta^g_{A_\mu}\,, \\
\left\{\delta^+,\delta^-\right\} &= \delta^g_{-\tau}\,, \qquad \left\{\delta^+_\mu,\delta^-\right\} = 0\,, \\
\left\{\delta^-_\mu,\delta^+\right\} &= 0\,, \qquad \left\{\delta^+_\mu,\delta^+_\nu\right\} = \delta^g_{-8g_{\mu\nu}\overline{\phi}}\,, \\
\left\{\delta^-_\mu,\delta^-_\nu\right\} &= \delta^g_{8g_{\mu\nu}\phi}\,, \qquad \left\{\delta^+_\mu,\delta^-_\nu\right\} = \delta^g_{-4iB_{\mu\nu}-4g_{\mu\nu}\tau} + \text{equations of motion}\,, \qquad (2.9)
\end{aligned}
$$

where δ^g_γ denotes a gauge transformation with parameter γ.

3. Quantization of the twisted theory

In order to quantize the action (2.2) one has to properly take into account the twisted generators $\delta^{\pm}, \delta^{\pm}_{\mu}$. This amounts to perform the gauge fixing procedure in a way compatible with the relevant global invariances of the action $S_{N=4}$. Two non-trivial aspects have therefore to be faced, namely: the nonlinearity of the twisted transformations of the fields (2.4)–(2.7) and the fact that the algebra of the generators $\delta^{\pm}, \delta^{\pm}_{\mu}$ closes on the space-time translations only on-shell and modulo field-dependent gauge transformations. The way out in quantizing this kind of model requires the use of the Batalin-Vilkovisky procedure, already successfully applied to the untwisted $N = 4$ theory by [3], and to $N = 2$ by [15, 16].

The first step is to introduce the BRST symmetry corresponding to the gauge invariance of the theory, i.e. $\delta^{g}_{\zeta} \to s$ and $\zeta \to c$, where c is the usual Faddeev-Popov (FP) ghost transforming as $sc = ic^2$. Following now [3, 15, 16], one extends the BRST operator into a new operator Q which turns out to be nilpotent on shell and which, together with the BRST charge s, collects all the other generators appearing in the algebra (2.9). Introducing thus a set of global ghosts $\omega^{\pm}, \varepsilon^{\pm\mu}, v^{\mu}$ associated to $\delta^{\pm}, \delta^{\pm}_{\mu}$, and to the space-time translations ∂_{μ}, the operator Q is found to be

$$Q = s + \omega^{+}\delta^{+} + \omega^{-}\delta^{-} + \varepsilon^{+\mu}\delta^{+}_{\mu} + \varepsilon^{-\mu}\delta^{-}_{\mu} + v^{\mu}\partial_{\mu} - (\omega^{+}\varepsilon^{+\mu} + \omega^{-}\varepsilon^{-\mu})\frac{\partial}{\partial v^{\mu}} . \tag{3.1}$$

From the requirement that Q carries FP charge $+1$, it follows that the global ghosts ω^{+}, ω^{-}, $\varepsilon^{+\mu}, \varepsilon^{-\mu}$ are commuting and have FP charge $+1$, while v^{μ} is anticommuting, with the same charge. Defining then the action of the operator Q on the Faddeev-Popov ghost c as [3, 15, 16]

$$\begin{aligned} Qc = ic^2 &+ \left(\omega^{+^2} - 4\varepsilon^{-^2}\right)\phi + \left(4\varepsilon^{+^2} - \omega^{-^2}\right)\overline{\phi} + \left(\omega^{+}\omega^{-} + 4\varepsilon^{+\mu}\varepsilon^{-}_{\mu}\right)\tau + \\ &+ 4i\varepsilon^{+\mu}\varepsilon^{-\nu}B_{\mu\nu} - \left(\omega^{+}\varepsilon^{+\mu} + \omega^{-}\varepsilon^{-\mu}\right)A_{\mu} , \end{aligned} \tag{3.2}$$

the following equations are seen to hold

$$\begin{aligned} QS_{N=4} &= 0 , \\ Q^2 &= 0 \text{ (modulo equations of motion)} . \end{aligned} \tag{3.3}$$

The next step is now to define the non-linear transformations of the fields $Q\Phi_i$ by coupling them to antifields Φ^{*}_{i}. This is done by introducing the external action

$$S_{\text{ext}} = \mathrm{tr} \int d^4x \; \Phi^{*}_{i} Q\Phi_i , \tag{3.4}$$

where, for a p-tensor field

$$\Phi^{*}_{i} Q\Phi_i = \frac{1}{p!}\Phi^{*\mu_1..\mu_p}_{i} Q\Phi_{i\mu_1..\mu_p} .$$

Field	c	ω^+	ω^-	$\varepsilon^{+\mu}$	$\varepsilon^{-\mu}$	v^μ
G-ch.	0	-1	1	1	-1	0
FP-ch.	1	1	1	1	1	1
Dim.	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1
Nature	a	c	c	c	c	a

Table 2: Quantum numbers of the ghost fields.

Source	A^*_μ	ξ^*	η^*	$\psi^*_{\mu\nu}$	$\chi^*_{\mu\nu}$	ψ^*_μ	χ^*_μ	ϕ^*	τ^*	$\overline{\phi}^*$	$B^*_{\mu\nu}$	H^*_μ	$H^*_{\mu\nu}$	c^*
G-ch.	0	-1	1	-1	1	-1	1	-2	0	2	0	0	0	0
FP-ch.	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-2
Dim.	3	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	3	3	3	3	2	2	4
nature	a	c	c	c	c	c	c	a	a	a	a	a	a	c

Table 3: Quantum numbers of the antifields.

In addition, due to the fact that the operator Q is nilpotent only on-shell, one has to introduce a further quadratic term in the antifields [3, 15, 16]:

$$S_{\text{quad}} = 4g^2\varepsilon^{+\mu}\varepsilon^{-\nu}\,\text{tr}\int d^4x\,\Big(\varepsilon_{\mu\nu\rho\lambda}A^{*\rho}H^{*\lambda} - \frac{1}{2}\left(B^{*\delta}_\nu H^*_{\mu\delta} - B^{*\delta}_\mu H^*_{\nu\delta}\right) - \\ - \varepsilon_{\mu\nu\rho\lambda}\psi^{*\rho}\chi^{*\lambda} + \frac{1}{2}\left(\psi^{*\delta}_\nu \chi^*_{\mu\delta} - \psi^{*\delta}_\mu \chi^*_{\nu\delta}\right)\Big)\,. \tag{3.5}$$

According to the Batalin-Vilkovisky procedure, we choose a Landau kind of gauge fixing term which takes the following form

$$S_{\text{gf}} = Q\,\text{tr}\int d^4x\,(\overline{c}\partial^\mu A_\mu) + 4g^2\varepsilon^{+\mu}\varepsilon^{-\nu}\,\text{tr}\int d^4x\varepsilon_{\mu\nu\rho\lambda}\partial^\rho\overline{c}H^{*\lambda}\,, \tag{3.6}$$

where the antighost $\overline{c}$, introduced by shifting the antifield A^*_μ as $A^*_\mu \to A^*_\mu + \partial_\mu\overline{c}$, is required to transform as

$$\begin{aligned} Q\overline{c} &= b + v^\mu\partial_\mu\overline{c}\,, \\ Qb &= (\omega^+\varepsilon^{+\mu} + \omega^-\varepsilon^{-\mu})\partial_\mu\overline{c} + v^\mu\partial_\mu b\,, \end{aligned} \tag{3.7}$$

b being the Lagrange multiplier implementing the Landau condition. Let us also display the quantum numbers of the ghosts and antifields (tables 2 and 3).
Finally, the complete gauge-fixed action Σ

$$\Sigma = S_{N=4} + S_{\text{ext}} + S_{\text{quad}} + S_{\text{gf}}\,, \tag{3.8}$$

turns out to obey the following Slavnov-Taylor identity

$$\mathcal{S}(\Sigma) = 0\,, \tag{3.9}$$

with

$$\mathcal{S}(\Sigma) = \operatorname{tr} \int d^4x \left(\frac{\delta\Sigma}{\delta\Phi_i^*} \frac{\delta\Sigma}{\delta\Phi_i} + (b + v^\mu \partial_\mu \overline{c}) \frac{\delta\Sigma}{\delta\overline{c}} - (\omega^+ \varepsilon^{+\mu} + \omega^- \varepsilon^{-\mu}) \frac{\delta\Sigma}{\delta v^\mu} + \right.$$
$$\left. + \Big((\omega^+ \varepsilon^{+\mu} + \omega^- \varepsilon^{-\mu}) \partial_\mu \overline{c} + v^\mu \partial_\mu b \Big) \frac{\delta\Sigma}{\delta b} \right) . \tag{3.10}$$

We underline that the Slavnov-Taylor identity (3.9) contains all the information concerning both the gauge and the $N = 4$ invariances of the classical starting action $S_{N=4}$. In other words, the present algebraic set up has allowed to obtain a gauge-fixed action which is compatible with the full set of defining symmetries. This point is of particular relevance, as allows one to analyse the quantum aspects of the theory by means of the powerful BRST cohomology techniques [17]. This will be the goal of the next section. However, before going any further, it is convenient to eliminate the ghost of the space-time translations v_μ by introducing the so-called reduced action $\widetilde{S}$

$$\Sigma \longrightarrow \widetilde{S} = \Sigma - v^\rho \frac{\partial\Sigma}{\partial v^\rho} - \operatorname{tr} \int d^4x \, b \partial^\mu A_\mu \, . \tag{3.11}$$

In terms of the action $\widetilde{S}$, the Slavnov-Taylor identity takes now the simplified form

$$\mathcal{S}\left(\widetilde{S}\right) = \operatorname{tr} \int d^4x \left(\frac{\delta\widetilde{S}}{\delta\Phi_i^*} \frac{\delta\widetilde{S}}{\delta\Phi_i} \right) = (\omega^+ \varepsilon^{+\mu} + \omega^- \varepsilon^{-\mu}) \Delta_\mu^{cl} \tag{3.12}$$

where the antifield $A^{*\mu}$ has to be replaced by the shifted antifield $\widehat{A}_\mu^* = A_\mu^* + \partial_\mu \overline{c}$, and Δ_μ^{cl} is given by

$$\Delta_\rho^{cl} = \operatorname{tr} \int d^4x \left(-\widehat{A}_\mu^* \partial_\rho A^\mu - H^{*\mu} \partial_\rho H_\mu - \frac{1}{2} B^{*\mu\nu} \partial_\rho B_{\mu\nu} - \tau^* \partial_\rho \tau - \right.$$
$$- \frac{1}{2} H^{*\mu\nu} \partial_\rho H_{\mu\nu} + \frac{1}{2} \psi^{*\mu\nu} \partial_\rho \psi_{\mu\nu} + \frac{1}{2} \chi^{*\mu\nu} \partial_\rho \chi_{\mu\nu} + \psi^{*\mu} \partial_\rho \psi_\mu +$$
$$\left. + \ \chi^{*\mu} \partial_\rho \chi_\mu + \xi^* \partial_\rho \xi + \eta^* \partial_\rho \eta - \phi^* \partial_\rho \phi - \overline{\phi}^* \partial_\rho \overline{\phi} + c^* \partial_\rho c \right) . \tag{3.13}$$

Notice that the above expression, being linear in the quantum fields, is a classical breaking and will not be affected by the quantum corrections. In addition, the Slavnov-Taylor identity (3.12) implies that the linearized Slavnov-Taylor operator $B_{\widetilde{S}}$ [17] defined as

$$B_{\widetilde{S}} = \operatorname{tr} \int d^4x \left(\frac{\delta\widetilde{S}}{\delta\Phi_i^*} \frac{\delta}{\delta\Phi_i} + \frac{\delta\widetilde{S}}{\delta\Phi_i} \frac{\delta}{\delta\Phi_i^*} \right) , \tag{3.14}$$

has the following property

$$B_{\widetilde{S}} B_{\widetilde{S}} = (\omega^+ \varepsilon^{+\mu} + \omega^- \varepsilon^{-\mu}) \partial_\mu \, , \tag{3.15}$$

which shows that $B_{\widetilde{S}}$ is nilpotent modulo a total derivative. As we shall see in detail in section 4, this feature will give rise to a set of very constrained descent equations which will allow to relate the full action of $N = 4$ to the gauge invariant polynomial $\operatorname{tr} \phi^2$, ϕ being the scalar field of the $N = 2$ vector multiplet.

4. The relationship between the action of $N = 4$ and tr ϕ^2

In order to establish the relationship between the action of $N = 4$ and tr ϕ^2 we proceed by analysing the cohomology of the operator $B_{\widetilde{S}}$ in the space of the integrated local functionals of the fields and their derivatives, with the same quantum numbers of the classical action. To this end it is worth recalling that the invariant action $S_{N=4}$ is uniquely fixed by the three generators δ^+_μ, δ^-_μ and δ^+. We can thus simplify our analysis setting to zero the global parameter ω^- corresponding to the scalar charge δ^-. Therefore, eqs. (3.12) and (3.15) take the following form

$$\mathcal{S}\left(\widetilde{S}\right) = \omega^+\varepsilon^{+\mu}\Delta^{cl}_\mu \,, \tag{4.1}$$

$$B_{\widetilde{S}}B_{\widetilde{S}} = \omega^+\varepsilon^{+\mu}\partial_\mu \,. \tag{4.2}$$

It should be observed from (4.2) that the operator $B_{\widetilde{S}}$ is not strictly nilpotent, as its square yields the space-time translations. This property is indeed a general feature of the supersymmetric gauge theories, following from the fact that the supersymmetric algebra closes on the space-time translations. In particular, equation (4.2) will give rise to a set of descent equations which will strongly constrain the cohomology of $B_{\widetilde{S}}$. As one can expect, this is related to the large number of generators which are encoded in the operator $B_{\widetilde{S}}$, reflecting the $N = 4$ structure of the theory.

In order to obtain the descent equations for the operator $B_{\widetilde{S}}$, let us start with the consistency condition

$$B_{\widetilde{S}} \int d^4x\, \Omega^0 = 0 \,, \tag{4.3}$$

where Ω^0 has the same quantum numbers of the classical lagrangian of $N = 4$, i.e. it is a local polynomial of dimension four and with vanishing FP and G-charge. Due to eq. (4.2), the integrated consistency condition (4.3) can be translated at the local level as

$$B_{\widetilde{S}}\Omega^0 = \partial^\mu\Omega^1_\mu \,, \tag{4.4}$$

where Ω^1_μ is a local polynomial of FP charge 1 and dimension 3. Applying now the operator $B_{\widetilde{S}}$ to both sides of (4.4) and making use of eq. (4.2), one obtains the condition

$$\partial^\mu\left(B_{\widetilde{S}}\Omega^1_\mu - \omega^+\varepsilon^+_\mu\Omega^0\right) = 0 \,, \tag{4.5}$$

which, due to the algebraic Poincaré lemma [17], implies

$$B_{\widetilde{S}}\Omega^1_\mu = \omega^+\varepsilon^+_\mu\Omega^0 + \partial^\nu\Omega^2_{[\nu\mu]} \,, \tag{4.6}$$

for some local polynomial $\Omega^2_{[\nu\mu]}$ antisymmetric in the Lorentz indices μ, ν and with FP charge 2. The procedure can be easily iterated, yielding the following set of descent equations

$$B_{\widetilde{S}}\Omega^0 = \partial^\mu\Omega^1_\mu \,,$$

$$
\begin{aligned}
B_{\widetilde{S}}\Omega^1_\mu &= \partial^\nu \Omega^2_{[\nu\mu]} + \omega^+\varepsilon^+_\mu \Omega^0 \,, \\
B_{\widetilde{S}}\Omega^2_{[\mu\nu]} &= \partial^\rho \Omega^3_{[\rho\mu\nu]} + \omega^+\varepsilon^+_\mu \Omega^1_\nu - \omega^+\varepsilon^+_\nu \Omega^1_\mu \,, \\
B_{\widetilde{S}}\Omega^3_{[\mu\nu\rho]} &= \partial^\sigma \Omega^4_{[\sigma\mu\nu\rho]} + \omega^+\varepsilon^+_\mu \Omega^2_{[\nu\rho]} + \omega^+\varepsilon^+_\rho \Omega^2_{[\mu\nu]} + \omega^+\varepsilon^+_\nu \Omega^2_{[\rho\mu]} \,, \\
B_{\widetilde{S}}\Omega^4_{[\mu\nu\rho\sigma]} &= \omega^+\varepsilon^+_\mu \Omega^3_{[\nu\rho\sigma]} - \omega^+\varepsilon^+_\sigma \Omega^3_{[\mu\nu\rho]} + \omega^+\varepsilon^+_\rho \Omega_{[\sigma\mu\nu]} - \Omega^3_{[\rho\sigma\mu]}\omega^+\varepsilon^+_\nu \,.
\end{aligned} \tag{4.7}
$$

It is interesting to observe that these equations are of an unusual type, as the cocycles with low FP charge appear in the equations of those with higher FP charge, turning the system (4.7) highly non trivial. We remark that the last equation for $\Omega^4_{[\mu\nu\rho\sigma]}$ is not homogeneous, a property which strongly constrains the possible solutions. To some extent, eq. (4.7) display a certain similarity with the descent equations in $N = 1$ superspace [18]. Actually, it is possible to solve the system (4.7) in a rather direct way by making use of the $N = 4$ structure. To this end, let us introduce the operator

$$
\mathcal{W}_\mu = \frac{1}{\omega^+}\left[\frac{\partial}{\partial\varepsilon^{+\mu}}, B_{\widetilde{S}}\right] , \tag{4.8}
$$

which obeys the relations

$$
\begin{aligned}
\left\{\mathcal{W}_\mu, B_{\widetilde{S}}\right\} &= \partial_\mu \,, \\
\left\{\mathcal{W}_\mu, \mathcal{W}_\nu\right\} &= 0 \,.
\end{aligned} \tag{4.9}
$$

This algebra is typical of topological quantum field theories [19]. In particular, as shown in [20], the decomposition (4.9) allows to making use of $\mathcal{W}_\mu$ as a climbing-up operator for the descent equations (4.7). It turns out in fact that the solution of the system is

$$
\begin{aligned}
\Omega^0 &= \frac{1}{4!}\mathcal{W}^\mu \mathcal{W}^\nu \mathcal{W}^\rho \mathcal{W}^\sigma \Omega^4_{[\sigma\rho\nu\mu]} \,, \\
\Omega^1_\mu &= \frac{1}{3!}\mathcal{W}^\nu \mathcal{W}^\rho \mathcal{W}^\sigma \Omega^4_{[\sigma\rho\nu\mu]} \,, \\
\Omega^2_{[\mu\nu]} &= \frac{1}{2!}\mathcal{W}^\rho \mathcal{W}^\sigma \Omega^4_{[\sigma\rho\mu\nu]} \,, \\
\Omega^3_{[\mu\nu\rho]} &= \mathcal{W}^\sigma \Omega^4_{[\sigma\mu\nu\rho]} \,,
\end{aligned} \tag{4.10}
$$

with $\Omega^4_{[\mu\nu\rho\sigma]}$ given by

$$
\Omega^4_{[\mu\nu\rho\sigma]} = \left(\omega^+\right)^4 \varepsilon_{\mu\nu\rho\sigma} \operatorname{tr}\phi^2 \,. \tag{4.11}
$$

From eqs. (4.10) the usefulness of the operator $\mathcal{W}_\mu$ becomes now apparent. Recalling thus that the cocycle Ω^0 has the same quantum numbers of the $N = 4$ lagrangian, the following relation holds

$$
\left(g\frac{\partial\widetilde{S}}{\partial g} - \varepsilon^{-\mu}\frac{\partial\widetilde{S}}{\partial\varepsilon^{-\mu}} \right)_{\omega^-=0} = -\frac{(\omega^+)^4}{96g^2}\varepsilon^{\mu\nu\rho\sigma}\mathcal{W}_\mu\mathcal{W}_\nu\mathcal{W}_\rho\mathcal{W}_\sigma \operatorname{tr}\int d^4x\, \phi^2 + B_{\widetilde{S}}\Xi^{-1} \,, \tag{4.12}
$$

for some irrelevant trivial cocycle Ξ^{-1}. Equation (4.12) follows from the observation that the derivatives of the Slavnov-Taylor identity (4.1) with respect to the coupling constant g and to the global ghost $\varepsilon^{-\mu}$ define integrated cohomology classes of the operator $B_{\widetilde{S}}$, namely

$$B_{\widetilde{S}} \left(\frac{\partial \widetilde{S}}{\partial g} \right)_{\omega^- = 0} = 0 \,, \tag{4.13}$$

and

$$B_{\widetilde{S}} \left(\frac{\partial \widetilde{S}}{\partial \varepsilon^{-\mu}} \right)_{\omega^- = 0} = 0 \,, \tag{4.14}$$

the first equation establishing the coupling constant g as a physical parameter of the theory.

We remark that the presence of the second term in the l.h.s. of eq. (4.12) is a consequence of setting to zero the ghost parameter ω^-, which implies that the classical breaking term in the r.h.s. of the Slavnov-Taylor identity (3.12) becomes independent from $\varepsilon^{-\mu}$. Furthermore, this term has a precise meaning, being related to the $\delta^-_\mu-$ invariance of the action (2.2). In fact, taking the equation (4.14) at zero antifields and ghosts, one gets

$$B_{\widetilde{S}} \left(\frac{\partial \widetilde{S}}{\partial \varepsilon^{-\mu}} \right)_{\Phi^*,\,\mathrm{ghosts}=0} = \delta^-_\mu S_{N=4} = 0 \,. \tag{4.15}$$

In particular, setting also $\varepsilon^{-\mu} = 0$ in the eq. (4.12), we finally obtain the key relationship

$$\left(g \frac{\partial \widetilde{S}}{\partial g} \right)_{\omega^-,\,\varepsilon^- = 0} \approx -\frac{(\omega^+)^4}{96 g^2} \varepsilon^{\mu\nu\rho\sigma} \mathcal{W}_\mu \mathcal{W}_\nu \mathcal{W}_\rho \mathcal{W}_\sigma \,\mathrm{tr} \int d^4x \, \phi^2 \,, \tag{4.16}$$

where $\approx$ means that the above equation holds modulo a trivial cocycle. We see therefore that, as announced, the full gauge-fixed action of $N = 4$ can be traced back to the gauge invariant polynomial tr ϕ^2. It should be noted that the introduction of the global ghosts $\omega^\pm, \varepsilon^\pm_\mu$ associated to the generators of $N = 4$ has to be seen as a useful device for carrying out a quantization procedure compatible with all invariances of the starting action. Since the physical observables of the theory, as for instance the chiral primary operators, are just required to be gauge invariant, the global ghosts have to be set to zero at the end.

The relation (4.16) represents the main result of the present work. It implies that many properties of the $N = 4$ action can be understood by looking at the gauge invariant polynomial $\mathrm{tr}\,\phi^2$, which plays the rôle of a kind of perturbative prepotential. In particular, following the same algebraic procedure adopted in the case of $N = 2$ [11], it is easy to prove that the anomalous dimension of the operator $\mathrm{tr}\,\phi^2$ vanishes to all orders of perturbation theory, implying that the β-function of the $N = 4$ can be at most of the order one-loop [11]. However, a direct inspection of the one-loop coefficient shows that β vanishes to all orders of perturbation theory. Consider in fact the general expression for the one-loop β-function of $N = 2$ [21]

$$\beta(g) = -\frac{g^3}{8\pi^2}\left(C_1 - hC_2\right), \tag{4.17}$$

where C_1 and C_2 are, respectively, the Casimir invariants of the representations of gauge and matter $N = 2$ multiplets, h being the number of matter multiplets. As is well known, the $N = 4$ SYM can be obtained by $N = 2$ when $h = 1$ and the matter multiplet belongs to the adjoint representation. Therefore $C_1 = C_2$, implying that $\beta(g) = 0$. This simple understanding of the perturbative ultraviolet finiteness of the theory provides a non-trivial example of how the properties of $N = 4$ can be in fact understood in terms of those of tr ϕ^2.

5. Conclusion

We have seen that the use of the twisted version of $N = 4$ allows to identify a subset of supercharges actually relevant for the non-renormalization aspects of the theory. This subset is given by the generators $\delta^+, \delta^+_\mu, \delta^-_\mu$, which uniquely fix the classical action. In addition, the vector charge δ^+_μ allows to solve easily the descent equations, leading us to relate the gauge-fixed action of $N = 4$ to the operator $\mathrm{tr}\,\phi^2$, see eq. (4.16). This result can be understood as an off-shell extension of the reduction formula discussed in [1], which plays an important rôle in the analysis of the non-renormalization theorems for the correlators of chiral primary operators [8, 9].

Moreover, as proven in [11], the Ward identity associated to the scalar supersymmetry implies the vanishing of the anomalous dimension of tr ϕ^2. This result may be considered as a first step towards an all-orders BRST algebraic analysis of the scaling properties of the above mentioned correlators.

It would be also useful to generalize eq. (4.16) to the case of a non-vanishing central charge for the $N = 4$ superalgebra, which would allow us to extend the proof of the ultraviolet finiteness of the theory to the spontaneously broken phase [22].

Acknowledgments

The Conselho Nacional de Desenvolvimento Científico e Tecnológico CNPq-Brazil, the Fundação de Amparo à Pesquisa do Estado do Rio de Janeiro (Faperj) and the SR2-UERJ are acknowledged for the financial support. A.T. thanks the Departamento de Física Teórica of the UERJ for the kind hospitality.

References

[1] K. Intriligator, *Bonus symmetries of $N = 4$ super Yang-Mills correlation functions via AdS duality*, Nucl. Phys. **B 551** (1999) 575 [hep-th/9811047].

[2] M.F. Sohnius and P.C. West, *Conformal invariance in $N = 4$ supersymmetric Yang-Mills theory*, Phys. Lett. **B 100** (1981) 245;
S. Mandelstam, *Light cone superspace and the ultraviolet finiteness of the $N = 4$ model*, Nucl. Phys. **B 213** (1983) 149;
P.S. Howe, K.S. Stelle and P.K. Townsend, *Miraculous ultraviolet cancellations in supersymmetry made manifest*, Nucl. Phys. **B 236** (1984) 125.

[3] P.L. White, *Analysis of the superconformal cohomology structure of $N = 4$ superyang-mills*, Class. and Quant. Grav. **9** (1992) 413.

[4] C. Montonen and D. Olive, *Magnetic monopoles as gauge particles?*, Phys. Lett. **B 72** (1977) 117.

[5] J. Maldacena, *The large-N limit of superconformal field theories and supergravity*, Adv. Theor. Math. Phys. **2** (1998) 231 [hep-th/9711200].

[6] E. D'Hoker, D.Z. Freedman, S.D. Mathur, A. Matusis and L. Rastelli, *Extremal correlators in the AdS/CFT correspondence*, hep-th/9908160.

[7] M. Bianchi and S. Kovacs, *Non-renormalization of extremal correlators in $N = 4$ sym theory*, Phys. Lett. **B 468** (1999) 102 [hep-th/9910016].

[8] K. Intriligator and W. Skiba, *Bonus symmetry and the operator product expansion of $N = 4$ super Yang-Mills*, Nucl. Phys. **B 559** (1999) 165 [hep-th/9905020].

[9] B. Eden, P.S. Howe and P.C. West, *Nilpotent invariants in $N = 4$ SYM*, Phys. Lett. **B 463** (1999) 19 [hep-th/9905085];
B. Eden, P.S. Howe, C. Schubert, E. Sokatchev and P.C. West, *Extremal correlators in four-dimensional SCFT*, Phys. Lett. **B 472** (2000) 323 [hep-th/9910150];
P.S. Howe, C. Schubert, E. Sokatchev and P.C. West, *Explicit construction of nilpotent covariants in $N = 4$ SYM*, Nucl. Phys. **B 571** (2000) 71 [hep-th/9910011];
B.U. Eden, P.S. Howe, E. Sokatchev and P.C. West, *Extremal and next-to-extremal N-point correlators in four-dimensional SCFT*, Phys. Lett. **B 494** (2000) 141 [hep-th/0004102].

[10] C. Vafa and E. Witten, *A strong coupling test of S-duality*, Nucl. Phys. **B 431** (1994) 3 [hep-th/9408074].

[11] A. Blasi, V.E.R. Lemes, N. Maggiore, S.P. Sorella, A. Tanzini, O.S. Ventura and L.C.Q. Vilar, *Perturbative β function of $N = 2$ super Yang-Mills theories*, J. High Energy Phys. **05** (2000) 039 [hep-th/0004048].

[12] J.P. Yamron, *Topological actions from twisted supersymmetric theories*, Phys. Lett. **B 213** (1988) 325.

JHEP01(2001)016

[13] E. Witten, *Topological quantum field theory*, *Comm. Math. Phys.* **117** (1988) 353.

[14] C. Lozano, *Duality in topological quantum field theories*, PhD thesis, Universidad de Santiago de Compostela, Spain 1999, hep-th/9907123;
J.M. F. Labastida and C. Lozano, *The Vafa-Witten theory for gauge group* SU(N), hep-th/9903172.

[15] F. Fucito, A. Tanzini, L.C.Q. Vilar, O.S. Ventura, C.A.G. Sasaki and S.P. Sorella, *Algebraic renormalization: perturbative twisted considerations on topological Yang-Mills theory and on $N = 2$ supersymmetric gauge theories*, hep-th/9707209;
A. Tanzini, O.S. Ventura, L.C.Q. Vilar and S.P. Sorella, *BRST cohomology of $N = 2$ super Yang-Mills theory in four dimensions*, *J. Phys.* **G 26** (2000) 1117.

[16] N. Maggiore, *Algebraic renormalization of $N = 2$ super Yang-Mills theories coupled to matter*, *Int. J. Mod. Phys.* **A 10** (1995) 3781 [hep-th/9501057]; *Off-shell formulation of $N = 2$ super Yang-Mills theories coupled to matter without auxiliary fields*, *Int. J. Mod. Phys.* **A 10** (1995) 3937.

[17] O. Piguet and S.P. Sorella, *Algebraic renormalization*, Monographs Series, M28, Springer Verlag, Berlin 1995.

[18] O. Piguet and K. Sibold, *Renormalization of $N = 1$ supersymmetrical Yang-Mills theories, 1. The classical theory*, *Nucl. Phys.* **B 197** (1982) 257;
Renormalization of $N = 1$ supersymmetrical Yang-Mills theories, 2. The radiative corrections, *Nucl. Phys.* **B 197** (1982) 272;
The anomaly in the slavnov identity for $N = 1$ supersymmetric Yang-Mills theories, *Nucl. Phys.* **B 247** (1984) 484;
Nonrenormalization theorems of chiral anomalies and finiteness in supersymmetric Yang-Mills theories,*Int. J. Mod. Phys.* **A 1** (1986) 913;
L.C.Q. Vilar, S.P. Sorella and C.A.G. Sasaki, *Superspace descent equations and zero curvature formalism of the four-dimensional $N = 1$ supersymmetric Yang-Mills theories*, *J. Math. Phys.* **40** (1998) 2735 [hep-th/9610220].

[19] F. Delduc, F. Gieres and S.P. Sorella, *Supersymmetry of the $D = 3$ Chern-Simons action in the Landau gauge*, *Phys. Lett.* **B 225** (1989) 367;
F. Delduc, C. Lucchesi, O. Piguet and S.P. Sorella, *Exact scale invariance of the Chern-Simons theory in the Landau gauge*, *Nucl. Phys.* **B 346** (1990) 313;
D. Birmingham, M. Blau, M. Rakowski and G. Thompson, *Topological field theory*, *Phys. Rep.* **209** (1991) 129.

[20] E. Guadagnini, N. Maggiore and S.P. Sorella, *Supersymmetric structure of four-dimensional antisymmetric tensor fields*, *Phys. Lett.* **B 255** (1991) 65;
S.P. Sorella, *Algebraic characterization of the Wess-Zumino consistency conditions in gauge theories*, *Comm. Math. Phys.* **157** (1993) 231 [hep-th/9302136].

[21] S. Weinberg, *The quantum theory of fields*, vol. 3, Cambridge University Press, Cambridge 2000.

[22] Work in progress.